Note: Some students like to use the "4-Step Method" to find the derivative of a function. Other students prefer to use the "derivative formula" from start to finish. Either approach may be used. YOU WILL RECEIVE NO CREDIT FOR APPLYING THE FORMULAS PRESENTED IN SECTIONS 4.2 AND BEYOND TO COMPLETE THESE QUESTIONS.

Watch your use of notation and "=" signs regardless of your approach
Write your limit notation correctly in step 4 if you use the "4 Step Method".

"DERIVATIVE FORMULA" means:
If \( y = f(x) \), then the derivative \( \frac{dy}{dx} \) is given by the formula
\[
\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]
provided the limit exists.

If you are using the derivative formula, substitute for \( f(x) \) and \( f(x+h) \) and simplify.

1. Find the derivative of \( f(x) = 4x^2 - 5 \) when \( x = 2 \). Use either the "4-Step" Method or the "Derivative Formula". (Hint: your answer will be a number)

   **STEP 1 (Point):** \( (2, 11) \)
   \( f(2) = 4 \cdot 2^2 - 5 = 16 - 5 = 11 \)

   **STEP 2 (Close Point):** \( (2+h, 11 + 16h + 4h^2) \)

   \[
f'(2+h) = 4(2+h)^2 - 5
   = 4(4 + 4h + h^2) - 5
   = 16 + 16h + 4h^2 - 5 = 11 + 16h + 4h^2
   \]

   **STEP 3 (Slope of the Secant):**

   \[
   \frac{(11 + 16h + 4h^2) - (11)}{h} = \frac{(2+h) - (2)}{h}
   \]

   \[
   \frac{16h + 4h^2}{h} = k(16 + 4h) = 16 + 4h
   \]

   **STEP 4 (limit(slope of the secant)):**

   \[
   \lim_{h \to 0} (16 + 4h) \to 0
   = 16 + 4 \cdot 0
   = 16
   \]

Write your answer to step 4 using two different derivative notations.

\[
f'(2) = 16
\]

October 2004
2. Demonstrate that formula for the derivative of \( f(x) = 4x^2 - 5 \) is \( 8x \). Use either the "4-Step" Method or the "Derivative Formula".

\[
\begin{align*}
\text{STEP 1:} & \quad (x, 4x^2 - 5) \\
\text{STEP 2:} & \quad (x + h, 4(x + h)^2 - 5) \\
& \quad 4(x + h)^2 - 5 \\
& \quad 4(x^2 + 2xh + h^2) - 5 \\
& \quad 4x^2 + 8xh + 4h^2 - 5 \\
\text{STEP 3:} & \quad \frac{4x^2 + 8xh + 4h^2 - 5 - (4x^2 - 5)}{h} = 8x + 4h \\
& \quad \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 8x + 4h \\
& \quad \lim_{h \to 0} \frac{4(x+h)^2 - 5 - (4x^2 - 5)}{h} = 8x + 4h \\
& \quad \lim_{h \to 0} \frac{4(x^2 + 2xh + h^2) - 5 - (4x^2 - 5)}{h} = 8x + 4h \\
& \quad \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 - 5 - (4x^2 - 5)}{h} = 8x + 4h \\
& \quad \lim_{h \to 0} \frac{8x + 4h}{h} = 8x \\
& \quad \lim_{h \to 0} \frac{\frac{f(h) - f(0)}{h}}{h} = 8x \\
& \quad \lim_{h \to 0} 8x + 4h = 8x + 0 = 8x \\
& \quad \frac{df}{dx} \bigg|_{x=2} = 8(2) = 16 \\
& \quad f'(2) = 8(2) = 16
\end{align*}
\]

Evaluate \( \frac{df}{dx} \bigg|_{x=2} \)

\( f'(2) = 8(2) = 16 \)

3. Find the derivative of the company's profit function, \( f(x) \), at \( x = 5 \). Use either the "4-Step" Method or the "Derivative Formula".

\( f(x) = 2x^2 - x + 8 \) million dollars, \( x \) years after 1990, from 1990 to 2000.

\[
\begin{align*}
\text{STEP 1:} & \quad (5, 53) \\
\text{STEP 2:} & \quad (5+h, 53 + 19h + 2h^2) \\
& \quad 2(5+h)^2 - 8 = 2(5^2 + 2 \cdot 5h + h^2) - 8 \\
& \quad 2(25 + 10h + h^2) - 8 = 50 + 20h + 2h^2 - 5 - h + 8 \\
\text{STEP 3:} & \quad \frac{50 + 20h + 2h^2 - 5 - h + 8}{h} \\
& \quad \lim_{h \to 0} \frac{f(5+h) - f(5)}{h} = \lim_{h \to 0} \frac{2(5+h)^2 -(5+h)+8 - 53}{h} = \lim_{h \to 0} \frac{2(5+10h + h^2) - (5+h)+8 - 53}{h} = \lim_{h \to 0} \frac{50 + 20h + 2h^2 - h + 8}{h} = \lim_{h \to 0} \frac{19h + 2h^2}{h} = \lim_{h \to 0} \frac{(9 + 2h)}{h} = \lim_{h \to 0} \frac{19 + 2h}{h} = 19 \\
& \quad f'(5) = 19
\end{align*}
\]
STEP 4: \( \lim_{h \to 0} (19 + 2h) = 19 \)
\[ f'(5) = 19 \]

Write your final answer in a sentence of practical interpretation.

In 1995, profit was increasing by \$19\text{ million} per year.

Find the percentage rate of change of \( f(x) \) at \( x = 5 \). Give units with your answer.

\[ \frac{f'(5)}{f(5)} \times 100 = \frac{19}{53} \times 100 = 35.849 \text{\% per year} \]

4. Show that the derivative of \( y = 4x^2 - 7x \) is \( \frac{dy}{dx} = 8x - 7 \). Use either the "4-Step" Method or the "Derivative Formula".

STEP 1: \( (x, 4x^2 - 7x) \)

STEP 2: \( (x+h, 4(x+h)^2 - 7(x+h)) \)

\[
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{4(x+h)^2 - 7(x+h) - (4x^2 - 7x)}{h}
\]

\[
= \lim_{h \to 0} \frac{4(x^2 + 2xh + h^2) - 7(x+h) - (4x^2 - 7x)}{h}
\]

\[
= \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 - 7x - 7h - 4x^2 + 7x}{h}
\]

\[
= \lim_{h \to 0} \frac{8xh + 4h^2 - 7h}{h}
\]

\[
= \lim_{h \to 0} (8x + 4h - 7)
\]

\[ = 8x - 7 \]

\[
\frac{dy}{dx} = 8x - 7
\]

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5. Write a formula for \( \frac{dy}{dx} \) given \( y = \sqrt{3x} \). Use either the "4-Step" Method or the "Derivative Formula".

\[
\lim_{h \to 0} \frac{\sqrt{3x+3h} - \sqrt{3x}}{h} = \lim_{h \to 0} \frac{\sqrt{3x+3h} + \sqrt{3x}}{\sqrt{3x+3h} + \sqrt{3x}} = \lim_{h \to 0} \frac{3x+3h-3x}{h(\sqrt{3x+3h} + \sqrt{3x})} = \lim_{h \to 0} \frac{3}{\sqrt{3x+3h} + \sqrt{3x}} = \frac{3}{2\sqrt{x}}
\]

\[
\frac{dy}{dx} = \frac{3}{2\sqrt{x}}
\]

5. Write a formula for \( \frac{dy}{dx} \) given \( y = \sqrt{x+5} \). Use either the "4-Step" Method or the "Derivative Formula".

\[
\lim_{h \to 0} \frac{\sqrt{x+h+5} - \sqrt{x+5}}{h} = \lim_{h \to 0} \frac{\sqrt{x+h+5} + \sqrt{x+5}}{\sqrt{x+h+5} + \sqrt{x+5}} = \lim_{h \to 0} \frac{(x+h+5) - (x+5)}{h(\sqrt{x+h+5} + \sqrt{x+5})} = \lim_{h \to 0} \frac{x+h+5 - x-5}{h(\sqrt{x+h+5} + \sqrt{x+5})} = \lim_{h \to 0} \frac{1}{h(\sqrt{x+h+5} + \sqrt{x+5})} = \frac{1}{\sqrt{x+5} + \sqrt{x+5}} = \frac{1}{2\sqrt{x+5}}
\]

\[
\frac{dy}{dx} = \frac{1}{2\sqrt{x+5}}
\]