1. \( R(w) \text{ cents} \) gives the cost of first class postage for a letter weighing \( w \) ounces.

a. Input units: ounces  Output units: cents  

Units on \( R'(w) \): cents per ounce

b. Give a sentence of interpretation for \( R'(1) = 40 \).

The cost of first class postage for a letter weighing 1 ounce is increasing by 40 cents per ounce.

c. If \( R(1) = 37 \), find and interpret in a sentence the PROC of \( R(w) \) at \( w = 1 \).

\[
\text{PROC} = \frac{R'(1)}{R(1)} \times 100 = \frac{40}{37} \times 100 = 108.108 \text{ cents per ounce}
\]

The cost of first class postage for a letter weighing 1 ounce is increasing by 108 cents per ounce.

d. Use \( R(1) \) and \( R'(1) \) to estimate \( R(3) \).

\[
R(3) \approx R(1) + 2 \cdot R'(1) = 37 + 40 + 40 = 1.17
\]

2. \( D(r) \) is the number of years it takes for an investment to double if \( r \% \) (percentage points) is the amount of interest compounded continuously.

a. Input units percentage pts  Output units years  

Units for \( D'(r) \) years per percentage pt

b. Give a sentence of interpretation for \( D(6) = 11.55 \).

The doubling time of an investment that pays 6% is 11.55 years.

c. Give a sentence of interpretation for \( D'(6) = -1.6 \).

The doubling time for an investment that pays 6% is decreasing by 1.6 years per (additional) percentage point (of interest).

c. Find the PROC for doubling time if \( r = 6 \). Give your answer in a sentence of practical interpretation.

\[
\frac{D'(6)}{D(6)} \times 100 = \frac{-1.6}{11.55} \times 100 = -13.853 \text{ percentage points per percent of interest.}
\]

October 2004
d. Would the time to double increase or decrease if $r$ decreases? Explain your answer.

If $r \uparrow$, $d.t \downarrow$ and if $r \downarrow$ then $d.t \uparrow$

It increases.

3. $P(m)$ is the median sale price (in thousands of dollars) of condominiums in a metropolitan area $m$ years after 1993

a. Input units _______ years _______ Output units _______ thousand $ _______

Units for $P(m)$ _______ thousand $ per year _______

b. Give a sentence of practical interpretation for $\frac{dP}{dm}_{m=6} = -8.2 \rightarrow P'(6) = -8.2$

In 1999, the median sale price of a condo was decreasing by $\$8,200$ per year.

c. If $P(6) = 120$, find and give a sentence of practical interpretation for the percentage rate of change (PROC) at $m = 6$.

$$\text{PROC} = \frac{P'(6)}{P(6)} \cdot 100 = \frac{-8.2}{120} \cdot 100 = -6.833 \% \text{ per year}$$

In 1999, the median sale price of a condo was decreasing by 6.833% per year.

d. Estimate $P(9)$.

$$P(9) = P(6) + 3 \cdot P'(6) = 120 - 8.2 - 8.2 - 8.2 = 95.4$$

$\$95,400$

4. $W(t)$ is the percentage of the popular vote that the candidate who won the election was receiving in a straw poll $(6-t)$ days before the election.

a. Input units _______ days _______ Output units _______ percentage vote _______

Units for $W(t)$ _______ percentage vote per day _______

October 2004
b. Give a sentence of practical interpretation for: \( \frac{dW}{dt} \bigg|_{t=4} = 2 \). \( W'(4) = 2 \) \( \frac{t - 4}{t - 0} = 2 \)

2 days before the election, the percentage of the popular vote received in a straw poll (by the candidate who won the election) was increasing by 2 percent per day.

If \( W(4) = 40 \), estimate the percent of the popular vote the eventual winning candidate received in a straw poll held 1 day before the election. (Hint: What will \( t \) be if \( 6 - t = 1 \)?)

\[ W(5) = 45 + 2 = 47\% \]

5. \( A(x) \) million is the number of passengers traveling on an airline during the \( x^{th} \) month of 1998

Input units: month | Output units: million passengers

Units for \( A'(x) \): million passengers per month

a. Interpret \( A'(9) = -1.28 \) million in a sentence.

In Sept., 1998, the no. of passengers traveling on an airline was decreasing by 1.28 million per month.

b. If \( A(9) = 50 \), estimate the number of passengers that flew on this airline during October.

\[ A(10) \approx 50 - 1.28 = 48.72 \text{ million} \]

6. \( P(x) \) (in thousands) is the projected population of 18 - 24 years olds in the state of South Carolina from 2005 to 2025, where \( x \) is the number of years since 1990.

a. Give a sentence of practical interpretation for each of the following notations:

\( P(22) = 408 \) \( \text{In 2012, the projected no. of South Carolinians between the ages of 18-24 is 408,000.} \)

\( P'(22) = -6.391 \) \( \text{In 2012, the projected no. of South Carolinians between the ages of 18-24 will be decreasing by 6391 per year.} \)

October 2004
b. Use the information from part a, to estimate \( P(23) \). Give your answer with units.

\[
P(23) \approx P(22) + P'(22) = 408 - 6.391 = 401.609 \text{ hours}.
\]

7. \( E(x) \) million students is the enrollment in public secondary schools in the US between 1940 and 2008, where \( x \) is the number of years since 1940.

Write each of the following statements using mathematical function notation or derivative notation. Each mathematical statement should be written as an equation.

a. The projected enrollment in 2008 is 14,400,000 students. \( 2008 - 1940 = 68 \)

\[
E(68) = 14.4
\]

b. In 1980, enrollment was declining by 0.19 million students per year. \( 1980 - 1940 = 40 \)

\[
E'(40) = -0.19
\]

c. In 1980, enrollment was 13.2 million students.

\[
E(40) = 13.2
\]

8. Suppose that \( G(v) \) miles per gallon gives the fuel efficiency of a car going \( v \) miles per hour. Write a sentence of practical interpretation for each of the following statements.

a. \( G(55) = 32.5 \). A car traveling 55 mph gets 32.5 mpg.

b. \( \frac{dg}{dv} \bigg|_{v=55} = -0.25 \). The fuel efficiency of a car traveling 55 mph is decreasing by 0.25 mpg for each additional mph (of velocity).

c. \( \frac{dG}{dv}[G(51)] = 0 \). The fuel efficiency of a car driving 51 mph is not changing.

d. \( G'(45) = 0.15 \). The fuel efficiency of a car traveling 45 mph is increasing by 0.15 mpg for each additional mph of velocity.