Use the following word bank in completing 1 - 3:

*inflection point, relative max, relative min, limiting value*

1. The graph in Figure 1 shows a(n) **relative min** and the graph in Figure 3 does not.

2. The graph in Figure 2 shows a(n) **inflection point** and the graph in Figure 1 does not.

3. The graph in Figure 4 shows a(n) **limiting value** and the graph in Figure 2 does not.

Use the names of the models to complete 4 - 5.

4. Name each of the functions graphed in Figures 1 through 4.

   a. Figure 1  **quadratic**  
   b. Figure 2  **cubic**  
   c. Figure 3  **exponential**  
   d. Figure 4  **logistic**

5. Even though you can often consider differences and/or the end behavior of models to help you decide which is best to fit to a certain set of data, the shape of the scatter plot plays a large part in that decision. Complete the following:

   a. No curvature indicates a **linear** model.

   b. A single concavity indicates that **exponential**, **quadratic**, or **logarithmic** models should be considered.

   c. An inflection point indicates that the **logistic** and **cubic** models should be considered.

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d. A limiting value indicates that the logistic or exponential models should be considered.

6. The monthly profit (in dollars) from the sale of Mike's Mobile Homes is given in the table below.

<table>
<thead>
<tr>
<th>Number of homes sold</th>
<th>7</th>
<th>8</th>
<th>11</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>19</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit (in dollars)</td>
<td>43,700</td>
<td>48,000</td>
<td>57,750</td>
<td>61,750</td>
<td>63,000</td>
<td>63,775</td>
<td>61,800</td>
<td>55,500</td>
</tr>
</tbody>
</table>

a. Give 2 characteristics of the scatter plot that indicate that a quadratic model is a reasonable choice.

The scatter plot is convex/downward and there is a local maximum.

b. Find a quadratic model. Call it \( P(x) \). Report your model to 3 decimal places. Define your model completely.

\[ f(x) = -246.253x^2 + 7918.856x + 389.507 \text{ dollars}, \]

where \( x \) is the number of homes sold \( 7 \leq x \leq 22 \)

c. This quadratic model is concave downward.

d. This continuous model should be interpreted \( \text{discretely} \) because you can't sell \( \frac{1}{2} \) of a mobile home.

e. If Mike's Mobile Homes sells 17 mobile homes in a month, the monthly profit will be \( \$63,043 \).

\[ f(17) \]

f. The process used to answer part e was interpolation/extrapolation because \( x = 17 \) is inside the domain.

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7. The table below gives the number of imported cars sold in the US between 1984 and 1994.

<table>
<thead>
<tr>
<th>Year</th>
<th>Imported Cars (thousands)</th>
<th>Year</th>
<th>Imported Cars (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>2440</td>
<td>1990</td>
<td>2400</td>
</tr>
<tr>
<td>1985</td>
<td>2840</td>
<td>1991</td>
<td>2040</td>
</tr>
<tr>
<td>1986</td>
<td>3245</td>
<td>1992</td>
<td>1940</td>
</tr>
<tr>
<td>1987</td>
<td>3200</td>
<td>1993</td>
<td>1780</td>
</tr>
<tr>
<td>1988</td>
<td>3000</td>
<td>1994</td>
<td>1735</td>
</tr>
<tr>
<td>1989</td>
<td>2700</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Align the data to years since 1980. Look at the scatter plot and give 2 reasons why a cubic model is an appropriate choice to try.

1st reason: There are two concentrations, one near 1987 & one near 1990.
2nd reason: There is no limiting value.

b. Find and completely define a cubic model for these data. Report your model to 3 decimal places.

The number of imported cars sold in the US is given by

\[ f(x) = 9.518x^3 - 282.494x^2 + 2469.806x - 3546.387 \]

where \( x \) is the number of years since 1980, \( 4 \leq x \leq 14 \).

c. Graph your model on the axes given. Label your axes.

![Graph](image)

\[ 1980 \quad 1990 \]

years since 1980

\[ 9500 \quad 6500 \]

#cars (thousands)

d. Mark the inflection point on the graph above. Explain what happened at the inflection point in the context of the problem situation.

The downward trend in the number of imported cars begins to reverse (at about 1990).

e. Use your unrounded model to predict the sales of imported passenger cars will be in 2002.

\[ 2002 - 1980 = 22 \]

September 2004 \[ y(22) = 15,413 \text{ thousand cars} \]
8. Population data for Iowa are given in the table below.

<table>
<thead>
<tr>
<th>Year</th>
<th>1980</th>
<th>1985</th>
<th>1987</th>
<th>1989</th>
<th>1991</th>
<th>1993</th>
<th>1995</th>
<th>1997</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thousand people</td>
<td>2910</td>
<td>2830</td>
<td>2770</td>
<td>2775</td>
<td>2790</td>
<td>2820</td>
<td>2840</td>
<td>2852</td>
</tr>
</tbody>
</table>

\[ y_1 = -2x^2 - 6x + 2910 \quad 0 \leq x \leq 7 \]
\[ y_2 = -1.191x^3 + 7.121x^2 - 75.464x + 3015.177 \quad 7 < x \leq 17 \]

A piecewise model is best for this data set. Align the data to 1980.

Divide the data into 2 sets with the 1987 \((x = 7)\) data point in both sets.

Use a quadratic model for the first data set and a cubic model for the second data set.

Write a completely defined piecewise model.

The population of Iowa is given by:

\[ f(x) = \begin{cases} 
-2x^2 - 6x + 2910 & 0 \leq x \leq 7 \\
-1.191x^3 + 7.121x^2 - 75.464x + 3015.177 & 7 < x \leq 17 
\end{cases} \]

thousand people

where \(x\) is the number of years since 1980.

9. The following table shows the number of people buying SUVs in October of the given year.

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thousand people</td>
<td>43</td>
<td>75</td>
<td>83</td>
<td>85</td>
<td>86</td>
<td>88</td>
<td>115</td>
<td>83</td>
<td>53</td>
</tr>
</tbody>
</table>

A piecewise model is best for this data set. Align the data to 1990.

Divide the data into 2 sets with the 1997 \((x = 7)\) data point in both sets.

Use a cubic model for the first data set and a linear model for the second data set.

Write a completely defined piecewise model.

The number of thousand people buying SUVs in October is given by:

\[ f(x) = \begin{cases} 
.930x^3 - 10.314x^2 + 36.777x + 44.339 & 0 \leq x < 7 \\
-15.5x + 223,167 & 7 \leq x \leq 11 
\end{cases} \]

where \(x\) is the number of years since 1990.