1. What happens to $f(x)$ as $x$ gets larger and larger or as $x$ gets smaller and smaller?

We abbreviate the statement "What happens to $f(x)$ as $x$ gets larger and larger?" with the notation $\lim \limits_{x \to \infty} f(x)$.

We abbreviate the statement "What happens to $f(x)$ as $x$ gets smaller and smaller?" with the notation $\lim \limits_{x \to -\infty} f(x)$.

Find each of the following:

a. \[ \lim \limits_{x \to \infty} f(x) = \bigcirc; \quad \lim \limits_{x \to -\infty} f(x) = \bigcirc \]

\[ \lim \limits_{x \to -\infty} f(x) = \bigcirc; \quad \lim \limits_{x \to \infty} f(x) = \bigcirc; \quad \lim \limits_{x \to \infty} f(x) = -\infty \]

c. \[ \lim \limits_{x \to -\infty} f(x) = \bigcirc; \quad \lim \limits_{x \to \infty} f(x) = \bigcirc \]

d. \[ \lim \limits_{x \to \infty} f(x) = \bigcirc; \quad \lim \limits_{x \to -\infty} f(x) = \bigcirc; \quad \lim \limits_{x \to \infty} f(x) = \bigcirc \]

2. For $f(x) = \frac{x - 1}{x^2 - x}$, numerically estimate the given limit. (Give answers to 3 decimal places.)

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{x } \to 1^+ & f(x) & \text{x } \to 1^- & f(x) & \text{x } \to 4^+ & f(x) & \text{x } \to 4^- & f(x) \\
\hline
x = 1.1 & .909 & x = .9 & 1.111 & x = 4.1 & .244 & x = 3.9 & .250 \\
\hline
x = 1.01 & .990 & x = .99 & 1.010 & x = 4.01 & .249 & x = 3.99 & .261 \\
\hline
x = 1.001 & .999 & x = .999 & 1.001 & x = 4.001 & .250 & x = 3.999 & .250 \\
\hline
x = 1.0001 & 1.000 & x = .9999 & 1.000 & x = 4.0001 & .250 & x = 3.9999 & .250 \\
\hline
\end{array}
\]

\[ \lim \limits_{x \to 1^+} f(x) = \bigcirc; \quad \lim \limits_{x \to 4^+} f(x) = \bigcirc \]

\[ \lim \limits_{x \to 1^-} f(x) = \bigcirc; \quad \lim \limits_{x \to 4^-} f(x) = \bigcirc \]

Verifications:

Verifications:

August 2004
c. \( \lim_{x \to \infty} f(x) \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \infty )</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>.01</td>
</tr>
<tr>
<td>1000</td>
<td>.001</td>
</tr>
<tr>
<td>10000</td>
<td>.0001</td>
</tr>
<tr>
<td>100000</td>
<td>.00001</td>
</tr>
</tbody>
</table>

\( \lim_{x \to \infty} f(x) = 0 \)

3. Use the graph to estimate the limits given.

a. \( \lim_{t \to 3} m(t) = 3 \)

b. \( \lim_{t \to 1} m(t) = 1 \)

c. \( \lim_{t \to 0} m(t) = \text{DNE} \)

d. \( \lim_{t \to \infty} m(t) = -\infty \)

e. \( \lim_{t \to -\infty} m(t) = 10 \)

4. Use the graph to estimate the limits given.

a. \( \lim_{r \to 3} f(r) = 3 \)

b. \( \lim_{r \to 9} f(r) = \infty \)

c. \( \lim_{r \to 0} f(r) = \text{DNE} \)

d. \( \lim_{r \to -4} f(r) = -4 \)

e. \( \lim_{r \to -\infty} f(r) = -\infty \)
5. State the 3 conditions for \( f(x) \) to be continuous at \( x = c \).

\[ f(x) = \begin{cases} 
\frac{6x}{x^2 - 6x} & \text{when } x < 6 \\
\frac{x^2 - 7}{x - 7} & \text{when } x \geq 6
\end{cases} \]

\[ f(6) = -1 \]

\[ \lim_{x \to 0} f(x) \]

a. Sketch a graph of the function for \(-5 \leq x \leq 20\), \(-15 \leq f(x) \leq 15\) on the axes shown below.

b. Use the graph to find the following limits:
\[ \lim_{x \to -6^-} f(x) = -\infty \quad \lim_{x \to -6^+} f(x) = -1 \quad \lim_{x \to 0^+} f(x) = \text{DNE} \]
\[ \lim_{x \to -6^-} f(x) = -\infty \quad \lim_{x \to -6^+} f(x) = -1 \quad \lim_{x \to 0^+} f(x) = \text{DNE} \]

5. Are there any horizontal asymptotes for this function? If so, give the equation(s) of the asymptote(s). If not, explain why not.

\[ f(x) = \text{DNE} \]

c. Discuss whether the function is continuous for the following inputs. If it is not continuous, tell why not.

\[ \bullet \quad x = 2 \quad \text{yes} \]
\[ \bullet \quad x = 6 \quad \text{no, there is a break in the graph at } x = 6 \]

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7. If a continuous function is used to model the following function descriptions, can it be used without restriction or must it be discretely interpreted? Explain.

a. \(Q(t)\) is your body temperature in degrees Celsius \(t\) hours after you take 500 mg of Tylenol.
   
   - Without restriction: \(t\) time and temperature are both measured continuously (no restrictions).

b. \(D(y)\) gives the on-line sales of DELL computers (in thousand computers) each year when \(y\) is the number of years after 2000.

   - Continuous with discrete interpretation because the on-line sales are measured once a year.
   \(x = 1 \Rightarrow\) the online sales in 2001, \(x = 2 \Rightarrow\) the online sales in 2002, etc.

C. \(R(t)\) is the revenue (in dollars) that Clemson makes selling \(t\) tickets to home football games.

   - Continuous with discrete interpretation because the input is restricted to whole numbers of tickets (we can't sell a fraction of a ticket).

D. \(C(r)\) is the number of children who register for summer day camp when the registration fee is \(r\) dollars.

   - Assuming the registration fee is a whole 
   \(C(r)\) is continuous with discrete interpretation

E. \(P(m)\) is the amount of property damage in millions of dollars caused by an earthquake of magnitude \(m\) in a certain west coast metropolitan area.

   - Assuming the machine measures the magnitude, there is no input restriction. The output, the amount of property damage also doesn't restrict the input. Therefore, \(P(m)\) is continuous without restriction.
8. The time it takes for an average 8- to 14-year-old athlete to swim the 100-meter freestyle can be modeled by the equation \( T(a) = -5.5a + 137 \) seconds where \( a \) is the age of the swimmer in years. The graph below is of \( T(a) \).

\[ \text{Average time to swim the 100-meter freestyle} \]

\[ \text{seconds} \]

\[ \text{age (in years)} \]

a. Can \( T(a) \) be used without restriction in this context or must it be discretely interpreted?
- \( a \) is the age \( \Rightarrow \) can be measured continuously
- \( T(a) \) is time \( \Rightarrow \) can be measured continuously
- \( T(a) \) \( \Rightarrow \) without restriction

b. If a young person can swim 100-meters freestyle in 1 minute and 20 seconds, what is the person's probable age?
\[ 80 = -5.5a + 137 \]
\[ a = 10.363 \text{ years} \]

9. Find the amount in an account after 16 months IF:

<table>
<thead>
<tr>
<th>Interest Rate</th>
<th>Compounded</th>
<th>Initial Deposit = ( P )</th>
<th>Amount after 16 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 7%</td>
<td>Semiannually, ( n=2 )</td>
<td>( $3900 = P )</td>
<td>( 3900 \left(1 + \frac{0.07}{2}\right)^2 = $4177.98 )</td>
</tr>
<tr>
<td>b. 2.1%</td>
<td>Monthly, ( n=12 )</td>
<td>( $2750 )</td>
<td>( 2750 \left(1 + \frac{0.021}{12}\right)^{12} = $2828.02 )</td>
</tr>
<tr>
<td>c. 10%</td>
<td>Quarterly, ( n=4 )</td>
<td>( $5000 )</td>
<td>( 5000 \left(1 + \frac{0.10}{4}\right)^{4} = $657.04 )</td>
</tr>
<tr>
<td>d. 3.75%</td>
<td>Continuously, Pert</td>
<td>( $6230 )</td>
<td>( 6230 \left(1 + \frac{0.0375}{12}\right)^{12} = $6549.42 )</td>
</tr>
</tbody>
</table>

e. How long does it take the investments in parts c and d to double?

\[ 10000 = 5000 \left(1 + \frac{0.10}{4}\right)^{t} \Rightarrow t = 7.018 \text{ yrs} \Rightarrow 7 \text{ yrs} + 3 \text{ months} \]

\[ 12460 = 6230 e^{0.0375t} \Rightarrow t = 18.484 \text{ yrs} \]