Under certain conditions, two functions may be combined to construct a meaningful function using addition, subtraction, multiplication or composition. Use the two functions given to construct another function.

1. \( S(x) \) is the number of students on the CAT bus where \( x \) is the number of hours after 7 am. 
   \( C(s) \) is the number of cars with tickets on them where \( s \) is the number of students on the bus.

   **Complete the table below.**

<table>
<thead>
<tr>
<th>Input description/units</th>
<th>( S(x) )</th>
<th>( C(s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>hours after 7am</td>
<td>number of students on the bus</td>
<td></td>
</tr>
<tr>
<td>Output description/units</td>
<td>number of students on the bus</td>
<td>number of cars with tickets</td>
</tr>
</tbody>
</table>

Use \( S(x) \) and \( C(s) \) to construct function \( T \) which gives the number of cars with parking tickets on them \( x \) hours after 7 am.

a. \( T \) may be constructed from \( S(x) \) and \( C(s) \) using **composition**.

b. The notation for the new function is \( T(x) = C(S(x)) \).

c. The input units for the new function are **hours**.

d. The output units for the new function are **cars**.

e. What does the new function calculate?

   \( C(S(x)) \) calculates the number of cars with tickets on them, \( x \) hours after 7 a.m.

2. The number of student tickets sold for a Clemson home basketball game can be modeled by \( S(w) \) tickets where \( w \) is the winning percentage of Clemson's team (entered as a decimal).

   The number of nonstudent tickets \( n(w) \) sold for a Clemson home basketball game can be modeled by \( N(w) \) hundred tickets where \( w \) is the winning percentage of Clemson's team (entered as a decimal).

   Use \( S(w) \) and \( N(w) \) to construct function \( T \) which gives the total number of tickets sold for a Clemson home basketball game, where \( w \) is the winning percentage.

   **Complete the table below.**

<table>
<thead>
<tr>
<th>Input description/units</th>
<th>( S(w) )</th>
<th>( N(w) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>winning % of CU team</td>
<td>winning % of CU team</td>
<td></td>
</tr>
<tr>
<td>Output description/units</td>
<td>student tickets</td>
<td>hundred non-student tickets</td>
</tr>
</tbody>
</table>

a. \( T \) may be constructed from \( S(w) \) and \( N(w) \) using **addition**.
b. The notation for the new function is \( T(w) = S(w) + 100N(w) \).

c. The input units for the new function are \( \text{percent} \).

d. The output units for the new function are \( \text{tickets} \).

e. What does the new function calculate?

\( T(w) \) calculates the total number of tickets sold to a Clemson home basketball game when the team has won \( w\% \) (as a decimal) of their home games.

3. \( F(t) \) is the number of female applicants to Clemson University in year \( t \).

\( M(t) \) is the number of male applicants to Clemson University in year \( t \).

Complete the table below.

<table>
<thead>
<tr>
<th>Input description/units</th>
<th>( F(t) )</th>
<th>( M(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output description/units</td>
<td>Year</td>
<td>Year</td>
</tr>
<tr>
<td>The number of female applicants to Clemson</td>
<td>The number of male applicants to Clemson</td>
<td></td>
</tr>
</tbody>
</table>

Use \( F(t) \) and \( M(t) \) to find function \( T \) which gives the total number of applicants to Clemson in year \( t \).

a. \( T \) may be constructed from \( F(t) \) and \( M(t) \) using addition.

b. The notation for the new function is \( T(t) = F(t) + M(t) \).

c. The input units for the new function are \( \text{year} \).

d. The output units for the new function are \( \text{applicants} \).

e. What does the new function calculate?

\( T(t) \) calculates the total number of applicants to Clemson University in year \( t \).

4. \( P(x) \) is the profit in thousand dollars of a company \( x \) years after it has been in business.

\( C(x) \) is the cost in thousand dollars of a company \( x \) years after it has been in business.

Complete the table below.

<table>
<thead>
<tr>
<th>Input description/units</th>
<th>( P(x) )</th>
<th>( C(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output description/units</td>
<td>number of years the company has been in business</td>
<td>number of years the company has been in business</td>
</tr>
<tr>
<td>thousand dollars of profit</td>
<td>thousand dollars of cost</td>
<td></td>
</tr>
</tbody>
</table>
a. Can \( P(x) \) and \( C(x) \) be used to construct a meaningful function? \( \text{yes} \)

b. If yes, give the notation of the new function. \( R(x) = P(x) + C(x) \)

c. If yes, input units of the new function: \( \text{years} \)

d. If yes, output units of the new function: \( \text{thousand dollars} \)

e. If yes, what does the new function calculate?
\( R(x) \) calculates the revenue in thousand dollars of a company \( x \) years after it has been in business.

5. \( P(x) \) is the revenue in thousand dollars of a company \( x \) years after 1990
\( C(x) \) is the cost in thousand dollars of a company in year \( x \).

Complete the table below.

<table>
<thead>
<tr>
<th>Input description/units</th>
<th>( P(x) )</th>
<th>( C(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output description/units</td>
<td>thousand dollars of revenue</td>
<td>thousand dollars of cost</td>
</tr>
<tr>
<td>Input description/units</td>
<td>the number of years after 1990</td>
<td>the year</td>
</tr>
</tbody>
</table>

a. Can \( P(x) \) and \( C(x) \) be used to construct a meaningful function? \( \text{no, the inputs are not the same, therefore this revenue and cost function can not be combined to find the profit.} \)

b. If yes, give the notation of the new function. 

c. If yes, input units of the new function: 

d. If yes, output units of the new function: 

e. If yes, what does the new function calculate?

6. The daily demand for beef can be modeled by \( D(p) = \frac{40}{1 + 0.03e^{0.4p}} \) million pounds when the price of the beef is \( p \) dollars per pound.

a. How can \( D(p) \) be used to construct a function for revenue? \( \text{By multiplying the demand by the price per pound} \)

b. Give the notation for the new function. \( R(p) = p \cdot D(p) \)

c. Input units of the new function: \( \text{dollars per pound} \)

d. Output units of the new function: \( \text{million dollars} \left(\text{dollars per pound} \times \text{million pounds}\right) \)

e. Equation for the daily revenue of beef: \( R(p) = p \cdot D(p) = \frac{40p}{1 + 0.03e^{0.4p}} \)
7. The population of West Virginia can be modeled as

\[ P(x) = \begin{cases} 
-23.514x + 3903.667 & \text{thousand people when } 85 \leq x < 90 \\
9.1x + 972.6 & \text{thousand people when } 90 \leq x \leq 93
\end{cases} \]

where \( x \) is the number of years since 1900.

a. Evaluate and write a sentence of practical interpretation for:

\[ P(85) = 1904.977 \]

In 1985, the population of West Virginia was 1,904,977.

\[ P(90) = 1791.6 \]

In 1990, the population of West Virginia was 1,791,600.

\[ P(93) = 1818.9 \]

In 1993, the population of West Virginia was 1,818,900.

b. Sketch a graph of \( P(x) \) between 1985 and 1993. Include a scale and units on your axes.