General Directions: Show all work analytically. When determining the convergence or divergence of a series, state the test (one on the handout) you use and analytically show in detail how you use it. If asked to approximate the sum and/or discuss a bound on the error for an alternating series, assume the alternating series satisfies the conditions of the Alternating Series Test. Summarize your conclusion in verbal sentence form.

For problems 1-4, determine if the series converges or diverges. If a series is geometric and convergent, find its sum. (9 points each)

1. \[ \sum_{n=1}^{\infty} \frac{2n^{3/2} + n}{\left(3n^2 + n\right)^2} \]

2. \[ \sum_{n=1}^{\infty} \cos \left( \frac{\pi n}{2} \right) \]
3. \[5 - 3 + \frac{9}{5} - \frac{27}{25} + \frac{81}{125} + \cdots\]

4. \[\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}\] (Use the integral test.)
For problems 5 and 6, determine whether the series is absolutely convergent, conditionally convergent, or diverges. (9 points each)

5. \[ \sum_{n=1}^{\infty} (-1)^n \frac{3}{4n^{2/3}} \]
6. \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} \] 

7. For the series \[ \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n + 1}{n!} \] find the smallest value of \( n \) such that the partial sum \( s_n \) approximates the actual sum with an error less than 0.01. (6 points)
8. For the power series

\[ \sum_{n=1}^{\infty} \frac{(x+1)^n}{3n 2^n} \]

a. What is the center? (2 points)

b. Analytically find the radius of convergence. (10 points)

c. Determine the interval of convergence. Be sure to include a check for convergence at the endpoints of the interval. (6 points)
9. For the function \( f(x) = \sin x \)
   a. Find the Maclaurin series expansion through the third non-zero term. Show all work. (6 pts)

   b. Use summation notation and the nth term to represent the series you found in part a. (4 pts)

   c. Use your series in part b to find a Maclaurin series to represent \( x \sin x \). (4 points)

   d. Use your answer in part c to find a Maclaurin series for \( \int x \sin x \, dx \). (6 points)

   e. Estimate \( \int_{0}^{1} x \sin x \, dx \) using the first three terms of the power series you found in part d. Determine a bound on the error. (6 points)