Note: For all integration problems, if the method of substitution is required be sure to write out $u$ and $du$. If integration by parts is required, be sure to write out $u$, $dv$, $du$, and $v$. For a definite integral give the exact value. For an improper integral indicate if it is convergent or divergent. All limits of integration should be written as exact values.

For problems 1-5, evaluate the integral.

1. (12 points) $\int \cos^3 \left( \frac{x}{3} \right) \, dx$

2. (12 points) $\int_0^{\pi/2} \tan x \, dx$
3. (12 points) \[ \int (2x + 1)^3 \, dx \]

4. (12 points) \[ \int_{\frac{\sqrt{3}}{3}}^{2\sqrt{3}} \frac{\sqrt{x^2 - 3}}{x} \, dx \]
5. (12 points) \( \int \arcsin(2x)dx \)

6. (12 points) Rewrite \( \frac{2x^3 + 2x^2 + 3x + 2}{x^2(x^2 + 1)} \) in terms of its partial fraction decomposition. (Determine the numerical values of the coefficients.)
7. (12 points) Set up and analytically find the exact value for the definite integral that gives the area of the region under the graph of \( f(x) = \frac{e^{3x}}{1 + e^{6x}} \) for \( x \geq 0 \). Shade the region on the graph below and state the answer as an exact value in a complete sentence.
Multiple Choice Problems 8-12 (4 points each): Each of the following multiple choice questions has only one solution. Circle the response that best answers the question. If your selection is correct, you will receive full credit (4 pts); if you do not circle any possible responses for a question, you will receive 0 points; and if you select an incorrect response, you will be penalized 1 point, i.e., you will receive a $-1$ for this question.

8. Find the value of the integral $\int \sin^2(5x)dx$.

A) $\frac{1}{2}x - \frac{1}{2}\sin(5x) + c$  
B) $\frac{1}{2}x + \frac{1}{10}\sin(5x) + c$  
C) $\frac{1}{2}x - \frac{1}{20}\sin(10x) + c$
D) $x - \frac{1}{10}\sin(10x) + c$  
E) none of the above.

9. Find the value of the integral $\int_1^2 x^3 \ln x dx$.

A) $8\ln 2 - \frac{3}{4}$  
B) $2\ln 2 - 1$  
C) $4\ln 2 - \frac{15}{16}$  
D) $\ln 4 - \frac{7}{8}$  
E) none of the above.

10. Evaluate the improper integral $\int_{-\infty}^{0} e^{3x} dx$.

A) 3  
B) $\frac{1}{3}$  
C) $\frac{-1}{3}$  
D) Divergent  
E) none of the above.
11. In the partial fraction decomposition \( \frac{x}{x^2 - 1} = \frac{A}{x + 1} + \frac{B}{x - 1} \) find the value of A.

A) -1/2        B) -2        C) -1         D) 1        E) none of the above.

12. When evaluating \( \int \sqrt{4x - x^2} \, dx \) by trigonometric substitution, the expressions \( \sqrt{4x - x^2} \) and \( dx \) are replaced by

A) \( \sqrt{4x - x^2} = 2 \cos(\theta) \) and \( dx = 2 \sin(\theta) \, d\theta \)
B) \( \sqrt{4x - x^2} = 2 \sin(\theta) \) and \( dx = 2 \cos(\theta) \, d\theta \)
C) \( \sqrt{4x - x^2} = 2 \cos(\theta) \) and \( dx = 2 \cos(\theta) \, d\theta \)
D) \( \sqrt{4x - x^2} = \cos(2\theta) \) and \( dx = \sin(2\theta) \, d\theta \)
E) none of the above.