Multiple Choice Problems 1-5 (4 points each): Each of the following multiple choice questions has only one solution. Circle the response that best answers the question. If your selection is correct, you will receive full credit (4 pts); if you do not circle any possible responses for a question, you will receive 0 points; and if you select an incorrect response, you will be penalized 1 point, i.e., you will receive a -1 for this question.

1. Which of the following is equal to $\ln 4$?

   (A) $\ln 3 + \ln 1$     (B) $\frac{\ln 8}{\ln 2}$     (C) $\int_1^4 \ln x \, dx$     (D) $\int_1^4 \frac{1}{t} \, dt$     (E) none of the above.

2. $\lim_{x \to 0} \frac{6^x - 2^x}{x} =$

   (A) $\ln 3$     (B) $\ln 4$     (C) 0     (D) 1     (E) the limit does not exist.

3. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of $x$?

   (A) $-\tan x$     (B) $-\cot x$     (C) $\cot x$     (D) $\tan x$     (E) $\csc x$
4. \( \int_{0}^{\ln 31} \frac{e^x}{2 + 2e^x} \, dx \)

(A) \ln 4 \hspace{1cm} (B) 2 \ln 4 \hspace{1cm} (C) 3 \ln 4 \hspace{1cm} (D) \frac{3}{2} \ln 4 \hspace{1cm} (E) \frac{1}{2} \ln 60

5. Express \((\ln 3) \log_3(2x) + 2 \ln(\cos x) - \ln(x^3 + 1)\) as a natural logarithm of a single quantity.

(A) \ln \left(\frac{2x + \cos^2 x}{x^3 + 1}\right) \hspace{1cm} (B) \ln \left(\frac{2x \cos^2 x}{x^3 + 1}\right) \hspace{1cm} (C) \ln \left(6x + 2 \cos x - (x^3 + 1)\right)

(D) \ln \left(\frac{(2x)^\ln 3 \cos^2 x}{x^3 + 1}\right) \hspace{1cm} (E) \text{cannot be written as a single term.
6. (5 pts) Find an algebraic expression for \( \cos \left( \arcsin \frac{2x}{3} \right) \). Show your work.

7. For \( y = 2x + \arctan(2x - 1) \)
   a. (5 pts) Find \( y' \).
   
   b. (4 pts) Write the equation of the tangent line to the curve at \( x=1 \).
   
   c. (3 pts) Find the intervals over which the function is increasing. Justify your conclusion.
8. (7 pts) Find the derivative for \( y = (2x)^x \).

9. A bacteria culture starts with 50 bacteria. The expression for the number of bacteria after \( t \) hours is \( f(t) = 50e^{\frac{\ln 8}{2}t} \).

   (a) (5 pts) How long does it take for the bacteria to double? Show your work. Give the answer as an exact value and use logarithmic properties to simplify.

   (b) (4 pts) At what rate does the bacteria grow after 2 hours. Show your work. Give the answer as an exact value and use logarithmic properties to simplify.
For problems 10-13, evaluate the integral. If the method of substitution is required, be sure to write out \( u \) and \( du \). For a definite integral give the exact value.

10. (8 points) \[ \int \frac{5 - x}{1 + 4x^2} \, dx \]

11. (8 points) \[ \int \frac{e^{3x} \arcsin(e^{3x})}{\sqrt{1 - e^{6x}}} \, dx \]
12. (8 points) \[ \int \frac{dx}{5x \sqrt[3]{\ln x}} \]

13. (10 points) \[ \int_{\pi/4}^{\pi/2} \frac{\sin(2x) \, dx}{1 + \cos^2(2x)} \]
For problems 14-16, evaluate the limit. If any limit is indeterminate, give its form. Show all work.

14. (6 points) \( \lim_{x \to 0} \frac{2 \arcsin(4x)}{7x} \)

15. (6 points) \( \lim_{x \to -\infty} \frac{3e^{5x} - 5e^{-5x}}{7e^{-5x} - 2e^{5x}} \)

16. (6 points) \( \lim_{x \to 1^+} \frac{2}{x^{1-x}} \)