For problems 1-4, find the derivative of the function. (7 points each)

1. \[ f(x) = \frac{1}{\ln x} + \ln \left( \frac{1}{x^2} \right) \]

2. \[ h(x) = \sqrt{1 - x^2} \arcsin(2x) \]

3. \[ y = \ln \left( \frac{x + 1}{\sqrt{x - 2}} \right) \]
4. \( y = (\sin x)^x \)

5. \( \int \frac{1 + e^{-x}}{e^2} \, dx \)

6. \( \int_{\frac{\pi}{2}}^{0} \frac{\sin x}{1 + \cos^2 x} \, dx \)
7. \[ \int 2^x \, dx \]

8. \[ \int \frac{\sec \theta \tan \theta}{1 + \sec \theta} \, d\theta \]

9. \[ \int \frac{3x + 2}{x^2 + 1} \, dx \]
For problems 10-11, evaluate the limit. Show all work. (6 points each)

10. \[ \lim_{{x \to \infty}} \frac{\ln x}{\ln \sqrt{x} + 10} \]

11. \[ \lim_{{x \to 0}} \frac{1 - e^x + x - \frac{1}{3}x^2}{x^2} \]

For the applications, 12-14, give the answers as exact values.

12. Let \( f(x) = \ln(\ln(x + 1)) \).
   a. Find \( f'(x) \). (7 point)

   b. Write the equation of the tangent line \( f(x) \) at \( x = e - 1 \). (7 points)
13. Find the maximum slope of \( f(x) = 3^{-x^2} \) as an exact value. Show all work. (8 points)
14. A bacteria culture grows at a rate proportional to the population size. The population is 100 bacteria initially and 1,600 after 4 hours. What is the instantaneous rate of population growth after 1 hour? Show all work and leave your answer as an exact value. Give the answer in a complete sentence. (8 points)