For problems 1-3, find the derivative of the function. (8 points each)

1. \( g(x) = 2^{-3x^2} \)

2. \( y = \ln \left( \frac{\tan x}{1 + x^2} \right) \)

3. \( f(x) = x \arctan(x^3) \)
For problems 4-8, evaluate the integral. If the method of substitution is required, be sure to write out u and du. For a definite integral give the exact value. (8 points each)

4. \[ \int \frac{1 + x - 3x^2}{x^2} \, dx \]

5. \[ \int \frac{e^x}{\sqrt{1 - e^{2x}}} \, dx \]

6. \[ \int e^{2x} \cos(e^{2x}) \, dx \]
7. \[
\int_{0}^{\sqrt[4]{3}} \frac{dx}{1 + 16x^2}
\]

8. \[
\int_{e}^{e^4} \frac{dx}{x\sqrt{\ln x}}
\]
For problems 9-11, evaluate the limit. Show all work. (4 points each)

9. \[ \lim_{{x \to 0}} \frac{e^x - e^{-x}}{\sin x} \]

10. \[ \lim_{{x \to \infty}} \arctan\left(e^x\right) \]

11. \[ \lim_{{x \to 0}} \frac{\ln(\sec^4 x)}{x^2} \]
12. Find the equation of the tangent line to the curve $y = \arccos(2x)$ where $x = 0$. Show your work. (8 points)

13. For the function $y = \frac{\ln x}{x}$, identify the coordinates of any relative extrema, determine if they are maximum or minimum values, and justify your answer analytically. (Give exact values.) State your answer in a complete sentence. (8 points)
14. A lettuce leaf collected from the salad bar at the college cafeteria contains $\frac{99}{100}$ as much carbon 14, ($^{14}C$) as a freshly cut lettuce leaf. How old is it? (Use 5700 years for the half-life of $^{14}C$.) Show your work and leave your answer as an exact value. Give the answer in a complete sentence. (8 points)

Bonus: (5 points)
Let $f(x) = 2\log_3(3x)$ and let $g$ be the inverse of $f$. Find the value of $g'(6)$. Show all work.