Questions 1 – 12 are multiple choice questions. Enter your answer for each question on the omr sheet. You will NOT be penalized for incorrect answers.

The omr sheet will not be returned so record your multiple choice responses on this test as well as the omr sheet.

In order to receive full credit for questions 17 - 22, you must

1. Show legible and logical (relevant) justification which supports your final answer. If you are testing a series for convergence or divergence, you must name the test used, state the conditions for that test and how they are met, and interpret the results of the test.

2. Use correct and complete notation.

3. Present the answer in a SUMMARY sentence.

If you are unaware of a question’s meaning, please raise your hand and your instructor will try to clear up any confusion.

NO CALCULATORS MAY BE USED ON THIS EXAM!

On my honor, I have neither given nor received inappropriate information during this exam.

SIGNATURE:___________________________________________

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MULTIPLE CHOICE_______ (out of 48)  FREE RESPONSE: _______ (out of 52)

TOTAL _________
Multiple Choice Questions. Choose the ONE correct answer for each question. Mark your answer on the scantron sheet. (48 pts)

1. If \( \sum_{n=1}^{\infty} a_n = 1 \), then \( \lim_{n \to \infty} a_n = \)
   a. 4
   b. 0
   c. 1
   d. 2
   e. cannot be determined

2. Find the radius of convergence of the series \( \sum_{n=1}^{\infty} \frac{n^2 x^n}{3^n} \).
   a. \( R = 0 \)
   b. \( R = 1 \)
   c. \( R = \frac{1}{3} \)
   d. \( R = 3 \)
   e. \( R = \infty \)

3. Identify the ONE correct statement.
   a. \( \{a_n\} = \left\{ \frac{1}{n} \right\} \) diverges; \( \{a_n\} = \left\{ (-1)^n \frac{1}{n} \right\} \) converges; \( \sum_{n=1}^{\infty} \frac{1}{n} \) diverges; \( \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \) converges
   b. \( \{a_n\} = \left\{ \frac{1}{n} \right\} \) diverges; \( \{a_n\} = \left\{ (-1)^n \frac{1}{n} \right\} \) diverges; \( \sum_{n=1}^{\infty} \frac{1}{n} \) converges; \( \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \) converges
   c. \( \{a_n\} = \left\{ \frac{1}{n} \right\} \) converges; \( \{a_n\} = \left\{ (-1)^n \frac{1}{n} \right\} \) converges; \( \sum_{n=1}^{\infty} \frac{1}{n} \) diverges; \( \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \) converges
   d. \( \{a_n\} = \left\{ \frac{1}{n} \right\} \) diverges; \( \{a_n\} = \left\{ (-1)^n \frac{1}{n} \right\} \) converges; \( \sum_{n=1}^{\infty} \frac{1}{n} \) diverges; \( \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \) diverges
   e. \( \{a_n\} = \left\{ \frac{1}{n} \right\} \) converges; \( \{a_n\} = \left\{ (-1)^n \frac{1}{n} \right\} \) converges; \( \sum_{n=1}^{\infty} \frac{1}{n} \) converges; \( \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \) diverges
4. Find a formula for the general term $a_n$, $n \geq 1$, of the sequence, assuming that the pattern of the first few terms continues.
\[
\begin{cases}
1, & 3, 9, 27, \\
\frac{1}{7}, \frac{3}{49}, \frac{9}{343}, \ldots
\end{cases}
\]

a. \[(-4)^{n-1}\]

b. \[(-1)^{n-1}\left(\frac{3}{7}\right)^{n-1}\]

c. \[(-1)^{n-1}\left(\frac{3}{7}\right)^{n-1}\]

d. \[(-1)^{n}\left(\frac{3}{7}\right)^{n-1}\]

e. \[(-1)^{n}\left(\frac{4}{7}\right)^{n-1}\]

5. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n \cdot 3^n}$.

a. \[R = 1\]

b. \[R = \frac{1}{3}\]

c. \[R = \frac{5}{3}\]

d. \[R = 2\]

e. \[R = 0\]

6. The sum of the geometric series $2 - 1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \ldots$ is

a. \[\frac{4}{3}\]

b. \[\frac{5}{4}\]

c. \[1\]

d. \[4\]

e. \[\frac{2}{3}\]
7. The sequence \( \{3r^n\} \) converges if and only if
   a. \( 0 < r < 1 \)
   b. \( |r| > 1 \)
   c. \( -1 < r \leq 1 \)
   d. \( |r| \leq 1 \)
   e. \( |r| < 1 \)

8. The series \( \sum_{n=1}^{\infty} \frac{n^4 + 15}{n^3} \) diverges by comparison with the series \( \sum_{n=1}^{\infty} b_n \). Find \( \sum_{n=1}^{\infty} b_n \).
   a. \( \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} n^5 \)
   b. \( \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^5} \)
   c. \( \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n} \)
   d. \( \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^2} \)
   e. \( \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} n \)

9. Find all values of \( x \) for which the series \( \sum_{n=1}^{\infty} \frac{(x-1)^n}{n} \) converges.
   a. \((0, 2)\)
   b. \([0, 2]\)
   c. \([-1, 1]\)
   d. \([0, 2]\)
   e. \([-1, 1]\)

10. If the radius of convergence of the power series \( \sum_{n=0}^{\infty} c_n x^n \) is \( R = 4 \), the radius of convergence of the power series \( \sum_{n=1}^{\infty} n c_n x^{n-1} \)
    a. equals \( \infty \)
    b. cannot be determined from the information given
    c. equals 0
    d. equals 1
    e. equals 4
11. \( s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n \) is the nth partial sum of a series defined as \( \sum a_n \)

The following statements concern the relationship of the sequence of partial sums \( \{s_n\} \) and the series \( \sum_{n=1}^{\infty} a_n \). Select the ONE false statement from the choices given.

a. If \( \lim_{n \to \infty} a_n = 0 \), then \( \sum_{n=1}^{\infty} a_n \) must be convergent.

b. If \( \sum_{n=1}^{\infty} a_n \) converges, then \( \{s_n\} \) must also be convergent.

c. If \( \sum_{n=1}^{\infty} a_n \) converges, then \( \lim_{n \to \infty} a_n = 0 \)

d. If \( \{s_n\} \) converges, then \( \sum_{n=1}^{\infty} a_n \) must also be convergent.

12. Complete the statement.

The series \( \sum_{n=1}^{\infty} 3^n(1-n) \)

a. converges and the sum is 6.

b. diverges.

c. converges and the sum is \( \frac{1}{2} \).

d. converges and the sum is 3.

e. converges, but the sum cannot be determined.

For All Written Problems:
- State the test that you are using (except in #13 when you are using the Integral Test)
- State the necessary conditions for the test that you are using and explain how they are met.
- Give your conclusion in a sentence.

13. Use the Integral Test to determine whether the series \( \sum_{n=3}^{\infty} \frac{1}{n \ln n} \) is convergent or divergent. (10 pts)

State the conditions necessary for use of the Integral Test and explain how they are met.
14. Determine whether \( \sum_{n=1}^{\infty} \left[ \frac{3^n + \left( \frac{1}{2} \right)^n}{n} \right] \) is convergent or divergent. \( (8 \text{ pts}) \)

You may use any test EXCEPT the Integral Test.

15. Find the interval of convergence of the series \( \sum_{n=1}^{\infty} \frac{(x)^n}{n4^n} \). (remember to test the endpoints) \( (10 \text{ pts}) \)
16. Determine whether $\sum_{n=1}^{\infty} (-1)^n \frac{n^2 5^n}{n!}$ is absolutely convergent, conditionally convergent, or divergent.

Support your conclusions analytically. Use correct notation. Write a summary sentence to support your conclusion. (8 pts)

17. Determine whether $\sum_{n=0}^{\infty} (-1)^n \frac{\sqrt{n}}{n+1}$ is absolutely convergent, conditionally convergent, or divergent.

Support your conclusions analytically. Use correct notation. Write a summary sentence to support your conclusion. (8 pts)
18. Find a power series representation (using $\sum_{n=d}^{\infty} c_n x^n$ notation) for the function $f(x) = \frac{1}{1 + x^2}$ and find the interval of convergence. (8 pts)